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Discussions of transient heat transfer to a water droplet on heated surfaces under atmospheric pressure

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Abstract—This paper discusses the transient heat transfer to a water droplet on heated surfaces considering a surface temperature change, in the contact period when the droplet is in contact with them. The initial surface temperature ranges from the lowest limit of the nucleate boiling region to the Leidenfrost point. Attention is paid to the two parts of the contact period: the waiting period before the onset of the first bubbling and the successive boiling period after this onset. Our proposal of transient heat transfer is based on some assumptions backed up by theoretical and experimental considerations—a theory of the first bubbling by Michiyoshi and a few experimental results from our previous papers and other works.

1. INTRODUCTION

WHEN A liquid droplet is placed on a heated surface, the liquid first comes into direct contact with the surface. In such a contact period the onset of the first bubbling in the droplet precedes the successive or vigorous boiling. Even if the surface temperature is very high, the droplet makes contact first and then bounces or floats [1]. We have observed changes in the appearance of a water droplet and in surface temperatures together with the time-averaged heat flux at the surface in the contact period, and we have also recognized the existence of a waiting period after which the first bubbling takes place [1].

Baumeister and Simon [2] developed a model for predicting the Leidenfrost temperature for discrete liquid drops, which takes into account that the drop–surface contact continued for 10 ms, where the time-averaged heat transfer coefficient at the heated surface was chosen as the one associated with the DNB point (departure from nucleate boiling) in a solution of the heat conduction equation in the heated wall. Nishio and Hirata [3] also predicted the Leidenfrost temperature by presuming that two successive stages of transient heat transfer had occurred in the contact period. In the former stage a transient heat conduction problem of sudden contact of two semi-infinite solids, initially at different temperatures, was applied to the waiting period with duration calculated by a theory of Han and Griffith [4]. In the latter stage, the heat transfer coefficient at the heated surface was assumed a value correlated by Kutateladze [5] for steady nucleate boiling. These two estimations, being solved under similar boundary and initial conditions, are mainly aimed at prediction techniques for the Leidenfrost temperature. Unfortunately neither model nor estimation has ever been made of such as a lower temperature range than the Leidenfrost temperature.

This paper intends to present such a new proposal of transient heat transfer to a droplet on heated surfaces in the contact period as it will be useful in the temperature range from the lowest limit of nucleate boiling region to the Leidenfrost point. The proposal is based on some assumptions: the surface temperature change is expressed by equations (1) and (3), the first bubbling occurs according to a theory by Michiyoshi [6], and a few experimental results [1, 7, 8] and other works [9–11] can be applied. In this proposal we postulate that the transient heat transfer in the waiting period takes place by conduction between a heated plate and the droplet first and then by convection in the droplet. The transient heat transfer coefficient in the waiting period will be presented.

2. TRANSIENT HEAT TRANSFER IN CONTACT PERIOD

2.1. *Summary of our proposal*

2.1.1. *In the waiting period before the onset of the first bubbling*; $0 \leqslant t \leqslant \tau_b$.

(a) After the droplet touches the surface, the surface temperature T_w falls instantaneously from its initial T_{w0} to interfacial contact temperature T_k, which is derived analytically by heat conduction theory, if we assume that two semi-infinite solids initially at different temperatures are suddenly brought into contact.

NOMENCLATURE

a	thermal diffusivity [$\mathrm{m^2\,s^{-1}}$]
c	specific heat of material [$\mathrm{J\,kg^{-1}\,K^{-1}}$]
g	acceleration of gravity [$\mathrm{m\,s^{-2}}$]
k	Boltzmann constant, $1.380\times10^{-23}\,\mathrm{J\,K^{-1}}$
m	ratio between surface heat fluxes, q_w/q_x
q	heat flux [$\mathrm{W\,m^{-2}}$]
r	instantaneous base radius of droplet in contact with the surface [m]
r^*	critical radius of embryonic bubble [m]
t	time [s]
x, y	coordinates, see Figs. 2 and 3
D_0	initial diameter of droplet [m]
E	heat energy stored in superheated liquid layer near the surface per unit surface area [$\mathrm{J\,m^{-2}}$]
L	length from lower edge along vertical plate [m]
P	pressure [Pa]
Pr	Prandtl number
T	temperature [°C]
T_B	bulk temperature of droplet [°C]
T_s	saturated temperature of water [°C]
X	thermophysical factor, $\sqrt{(c\rho\lambda/\pi)}$ [$\mathrm{J\,K^{-1}\,m^{-2}\,s^{-1/2}}$].

Greek symbols

α	heat transfer coefficient [$\mathrm{W\,m^{-2}\,K^{-1}}$]
β	volumetric coefficient of expansion [$\mathrm{K^{-1}}$]
δ	penetration depth perpendicular to surface, see Fig. 2 [m]
θ_w	difference between surface temperature measured by thermocouple and saturation temperature [K]
θ_{wc}	difference between surface temperature derived theoretically from Θ_w with τ_0 and saturation temperature [K]
Θ	$T-T_s$
λ	thermal conductivity [$\mathrm{W\,m^{-1}\,K^{-1}}$]
Λ	latent heat of evaporation referred to one molecule of water [$\mathrm{J\,mol^{-1}}$]
ρ	density [$\mathrm{kg\,m^{-3}}$]
σ	surface tension of water [$\mathrm{N\,m^{-1}}$]
τ_c	time at which purely one-dimensional heat conduction is terminated [s]
τ_r	first-order vibration period of droplet, see ref. [4] [s]
τ_t	contact period [s]
τ_0	time-constant of thermocouple [s].

Subscripts

b	onset of the first bubbling
k	interfacial contact
t	time-averaged, except for τ_t
w	surface or surface material
x	one-dimensional
L	water
0	initial.

The interfacial contact temperature T_k can be obtained [12] as follows:

$$T_k = (T_{w0}\sqrt{(c\rho\lambda)_w}+T_{L0}\sqrt{(c\rho\lambda)_L})/(\sqrt{(c\rho\lambda)_w}+\sqrt{(c\rho\lambda)_L}).$$

Putting

$$X_w = \sqrt{(c\rho\lambda)_w}/\sqrt{(\pi)}$$

and

$$X_L = \sqrt{(c\rho\lambda)_L}/\sqrt{(\pi)}$$

for the thermophysical properties of the surface material and pure water, respectively, and $X_R = X_w/X_L$ for their ratio, and also remembering the initial temperature T_{L0} of a water droplet at about 20°C in the experimental works [7] then

$$T_k = (T_{w0}X_R+20)/(X_R+1) \qquad (1)$$

$$X_R = X_w/X_L. \qquad (2)$$

(b) After (a), the surface temperature $\Theta_w(=T_w-T_s)$ drops from $\Theta_k(=T_k-T_s)$ primarily by convection within the droplet. Let us suppose that Θ_w is described by equation (3), where the coefficient K and exponent n are positive constants in the waiting period

$$\Theta_w = \Theta_k(1-Kt^n);\quad \Theta_k = T_k-T_s \text{ (see Fig. 1)} \qquad (3)$$

$$\mathrm{d}\Theta_w/\mathrm{d}t = \mathrm{d}T_w/\mathrm{d}t = -nK\Theta_k t^{n-1} < 0. \qquad (3)'$$

At the onset of first bubbling, $t=\tau_b$, the following equation holds:

$$\Theta_b = \Theta_k(1-K\tau_b^n). \qquad (4)$$

Other works [9–11] using a stainless steel surface and a thermometer with a short time constant showed similar temperature changes to equations (1) and (3) (see Fig. 8(c)).

(c) When a droplet is cooling a heated plate through its contact surface area, the contact area is changing during the contact period as described in the previous paper [1], hence it is troublesome to evaluate exactly such a complicated heat conduction problem within the heated plate. We simply assume that a practical heat flux q_w at the contact surface is given by mq_x, where q_x is the surface heat flux derived analytically from the one-dimensional heat conduction equation of the semi-infinite solid wall when its surface temperature changes in the same manner as Θ_w in equation (3) and m is a positive constant in the waiting period.

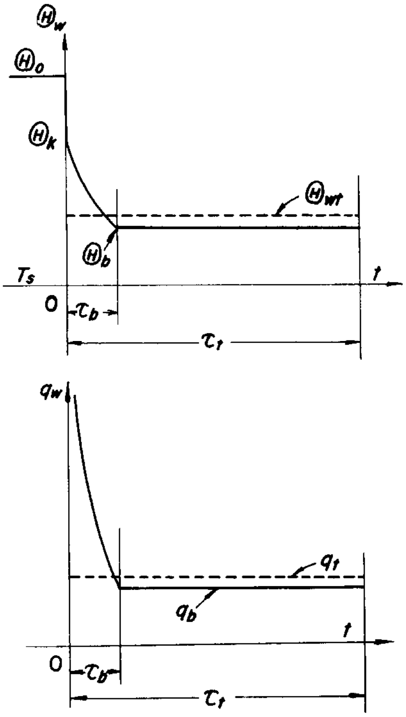

FIG. 1. Assumptions of change in Θ_w and q_w with time t.

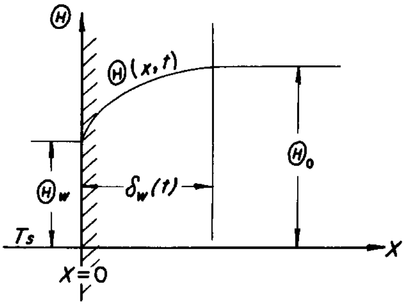

FIG. 2. Temperature distribution of penetration depth within heating plate.

When a semi-infinite solid initially at uniform temperature Θ_0 loses heat from its free surface, a temperature field is built up within the body as shown schematically in Fig. 2. This temperature field comprises a region wherein the local temperature differs from Θ_0.

The local temperature $\Theta(x, t)$ within a penetration depth $\delta_w(t)$ might be a form of quadratic equation in x. The coefficients of the equation of Θ can be obtained by the boundary conditions; $\Theta = \Theta_w$ at $x = 0$, and $\Theta = \Theta_0 =$ constant. $\partial\Theta/\partial x = 0$ at $x = \delta_w(t)$. Thus

$$\Theta(x, t) = \Theta_w + 2(\Theta_0 - \Theta_w)(x/\delta_w) + (\Theta_w - \Theta_0)(x/\delta_w)^2. \quad (5)$$

On the other hand, the one-dimensional heat conduction equation with constant thermophysical properties is

$$\partial\Theta/\partial t = a_w(\partial^2\Theta/\partial x^2).$$

It can be integrated from $x = 0$ to $\delta_w(t)$

$$\int_0^{\delta_w} (\partial\Theta/\partial t)\,dx = \int_0^{\delta_w} a_w(\partial^2\Theta/\partial x^2)\,dx$$

thus the equation becomes [13]

$$d\left[\int_0^{\delta_w} \Theta(x, t)\,dx - \Theta_0\delta_w(t)\right]\Big/ dt = -a_w\,\partial\Theta(0, t)/\partial x.$$

Substitution of equation (5) into the above equation yields

$$2\delta_w(d\delta_w/dt) - [2\delta_w^2/(\Theta_0 - \Theta_w)](d\Theta_w/dt) = 12a_w.$$

Using the initial condition $\delta_w(t) = 0$ at $t = 0$ and also equations (3) and (3)′, $\delta_w(t)$ can be solved

$$\delta_w(t) = \sqrt{}[12a_w t\{b_1^2 + (2a_1b_1t^n)/(n+1) + (a_1^2t^{2n})/(2n+1)\}]/(a_1t^n + b_1) \quad (6)$$

where $a_1 = K\Theta_k$, $b_1 = \Theta_0 - \Theta_k$. With equations (5) and (6), the heat flux q_x at the surface is

$$\begin{aligned} q_x &= \lambda_w(\partial\Theta/\partial x)_{x=0} = 2\lambda_w(\Theta_0 - \Theta_w)/\delta_w \\ &= \{\lambda_w(a_1t^n + b_1)^2\}/\sqrt{}[3a_w t\{b_1^2 + (2a_1b_1t^n)/(n+1) + (a_1^2t^{2n})/(2n+1)\}]. \end{aligned}$$

Hence the practical heat flux q_w and its differential form dq_w/dt can be written as

$$q_w = mq_x = \sqrt{}(\pi/3)\{mX_w(a_1t^n + b_1)^2\}/[t^{1/2}\sqrt{}\{b_1^2 + (2a_1b_1t^n)/(n+1) + (a_1^2t^{2n})/(2n+1)\}]; \quad 0 \leqslant t < \tau_b \quad (7)$$

$$\frac{dq_w}{dt} = \left(\frac{\pi}{3}\right)^{1/2} mX_w \frac{(a_1t^n + b_1)\left\{\left(\dfrac{2n-1}{2n+1}\right)a_1^3t^{3n} + \left(\dfrac{5n-3}{n+1}\right)a_1^2b_1t^{2n} + (4n-3)a_1b_1^2t^n - b_1^3\right\}}{2t^{3/2}\left(b_1^2 + \dfrac{2a_1b_1t^n}{n+1} + \dfrac{a_1^2t^{2n}}{2n+1}\right)^{3/2}}. \quad (7)'$$

Since $q_w = q_b$ at $t = \tau_b$ in equation (7), it follows that

$$q_b = \sqrt{}(\pi/3)\{mX_w\}\frac{(a_1\tau_b^n + b_1)^2}{\tau_b^{1/2}\sqrt{\left(b_1^2 + \dfrac{2a_1b_1\tau_b^n}{n+1} + \dfrac{a_1^2\tau_b^{2n}}{2n+1}\right)}}; \quad t = \tau_b \quad (8)$$

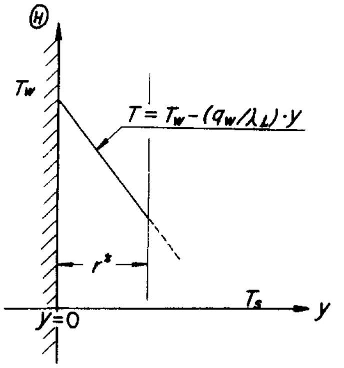

FIG. 3. Temperature distribution within the thickness r^*.

(d) As for the onset of first bubbling, Michiyoshi [6] proposed that when heat energy E, which is necessary to superheat a liquid layer of thickness r^* over the saturation temperature, attains its maximum value, nucleation can occur and the formation of new bubbles can proceed in the liquid layer. The thickness r^* is so small that the distribution of temperature within r^* may be linear, the temperature gradient may be equal to $-q_w/\lambda_L$ and a sort of heat capacity $c_L\rho_L$ is a constant, see Fig. 3. Consequently, the heat energy E is given by the following equation:

$$E = \int_0^{r^*} c_L\rho_L(T-T_s)\,\mathrm{d}y$$
$$= c_L\rho_L\{(T_w-T_s)r^* - (q_w/2\lambda_L)(r^*)^2\}. \quad (9)$$

On the other hand, the thickness r^* is assumed to be the critical radius of the embryonic bubble [6] and is given by

$$r^* = 2\sigma/[P[\exp\{(T_w-T_s)\Lambda/(kT_sT_w)\}-1]]. \quad (10)$$

The value of r^* is only a function of T_w, because other factors in the above equation are constant when P is constant. So the differential of equation (10) becomes

$$\mathrm{d}r^*/\mathrm{d}T_w = -(2\sigma/P)(\Lambda/kT_w^2)\exp\{(T_w-T_s)$$
$$\times\Lambda/(kT_sT_w)\}/[\exp\{(T_w-T_s)\Lambda/(kT_sT_w)\}-1]^2. \quad (11)$$

Since T_w is a function of t by equation (3), r^* and $\mathrm{d}r^*/\mathrm{d}T_w$ can only be a function of time t. Then, the heat energy E in equation (9) could be differentiated with respect to t

$$\frac{\mathrm{d}E}{\mathrm{d}t} = c_L\rho_L\left\{(T_w-T_s)\frac{\mathrm{d}r^*}{\mathrm{d}t} + r^*\frac{\mathrm{d}T_w}{\mathrm{d}t}\right.$$
$$\left. - \frac{q_w r^*}{\lambda_L}\frac{\mathrm{d}r^*}{\mathrm{d}t} - \frac{(r^*)^2}{(2\lambda_L)}\frac{\mathrm{d}q_w}{\mathrm{d}t}\right\}. \quad (12)$$

Setting $\mathrm{d}E/\mathrm{d}t = 0$, we can obtain the condition of $E_{\max}$. This is the condition at the onset of the first bubbling by Michiyoshi [6], where $q_w = q_b$ and $T_w = T_b$ or $\Theta_w = \Theta_b$ at $t = \tau_b$.

Upon substituting equations (3), (3)′, (7), (7)′, (10) and (11) into equation (12), the condition of the onset is written as

$$\frac{q_b\left\{\dfrac{2n-1}{2n+1}a_1^3\tau_b^{3n} + \dfrac{5n-3}{n+1}a_1^2b_1\tau_b^{2n} + (4n-3)a_1b_1^2\tau_b^n - b_1^3\right\}}{2\left(b_1^2 + \dfrac{2a_1b_1\tau_b^n}{n+1} + \dfrac{a_1^2\tau_b^{2n}}{2n+1}\right)(a_1\tau_b^n + b_1)}$$
$$= \frac{2B\Lambda n a_1\tau_b^n}{k(B-1)T_b^2}\left[\frac{P(B-1)\lambda_L}{2\sigma}\left\{\Theta_b - \frac{kT_b^2(B-1)}{\Lambda B}\right\} - q_b\right] \quad (13)$$

where $B = \exp\{(T_b-T_s)\Lambda/(kT_bT_s)\}$.

Two factors

$$A_0 = (2B\Lambda)/\{k(B-1)T_b^2\}$$

and

$$A_1 = \{P(B-1)\lambda_L\}/(2\sigma)\times\{\Theta_b - kT_b^2(B-1)/(\Lambda B)\}$$

on the right-hand side of equation (13) might be a function of T_b or Θ_b, because B is a function of T_b and other factors, k, P, T_s, Λ, λ_L and σ are constant or nearly constant. It is useful to illustrate the relation between the two factors and Θ_b for water under atmospheric pressure as shown in Fig. 4. These relations could be expressed approximately by equations (14) and (15), respectively, which are shown by solid lines in Fig. 4

$$A_0 \cong 1.53\Theta_b^{-0.819} \quad (14)$$

$$A_1 \cong 281\,\Theta_b^{3.05}. \quad (15)$$

Hence a simpler expression of equation (13) is

$$\frac{q_b\left\{\dfrac{2n-1}{2n+1}a_1^3\tau_b^{3n} + \dfrac{5n-3}{n+1}a_1^2b_1\tau_b^{2n} + (4n-3)a_1b_1^2\tau_b^n - b_1^3\right\}}{2\left(b_1^2 + \dfrac{2a_1b_1\tau_b^n}{n+1} + \dfrac{a_1^2\tau_b^{2n}}{2n+1}\right)(a_1\tau_b^n + b_1)}$$
$$= 1.53\Theta_b^{-0.819}na_1\tau_b^n(281\Theta_b^{3.05} - q_b). \quad (16)$$

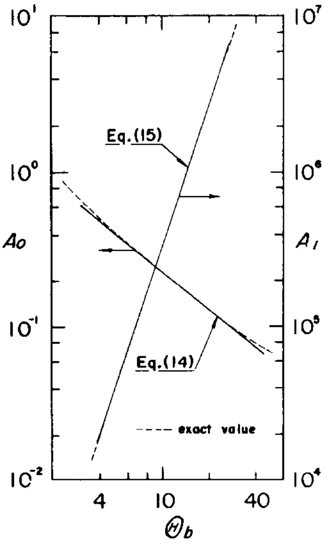

FIG. 4. The relation among A_0, A_1 and Θ_b.

2.1.2. *In the successive or vigorous boiling period*; $\tau_b < t < \tau_t$.

(a) Although surface temperature T_w showed a wavy value [1, 7, 9–11], T_w or Θ_w could be put as a constant, T_b or Θ_b (see Fig. 1).

(b) In general, there are some problems in measuring the surface temperature change by a thermocouple with a time constant τ_0 [14]. The longer the contact period is and the slower the surface temperature change is, the smaller the discrepancy between the theoretical value and the measured value at the end of the contact period. Even if a contact period were only four times as long as the time constant τ_0 and the surface temperature changed downwards stepwisely from Θ_0 to Θ_b, the value of $(\Theta_0 - \theta_w)$ measured by the thermocouple would have a value of 98.2% of $(\Theta_0 - \Theta_b)$ at the end of the contact period; in other words $\theta_{w\,\min} \cong \Theta_b$, in which $\theta_{w\,\min}$ represents minimum value in θ_w. Since the time constant τ_0 of the thermocouple used in our experiment is about 15 ms and the shortest τ_t is 71 ms as shown in Table 1, it can be assumed that

$$\Theta_b \cong \theta_{w\,\min}. \tag{17}$$

As can be seen in Fig. 8, the assumption is suitable.

(c) Heat flux q_w is also kept constant and equal to q_b (see Fig. 1). In this period we also suppose such a quasi-static change in q_w as in Θ_w or T_w.

2.1.3. *Experimental results.* Though we correlated Θ_0 and q_t with θ_{wt} [1], one of the key temperatures in the present proposal could be $\theta_{w\,\min}$ as mentioned above. So it is interesting to correlate the relation among Θ_0, q_t and $\theta_{w\,\min}$ from our experimental data, which were taken for a water droplet under atmospheric pressure. In arranging them attention is paid to select the data in which the surface temperature drops monotonously to $\theta_{w\,\min}$ as shown in Fig. 8(b) of the previous paper [7]. These monotonous tem-

Table 1. Examples of numerical results by the present proposal

	Copper plate	Brass plate	Carbon steel plate	Stainless steel plate	Units	Remarks ($D_0 = 3.29$ mm)
Θ_0	20	30	35	50	K	
Θ_k	15.65	20.75	22.67	27.35	K	eqns (1) and (2)
Θ_{wt}	11.94	10.86	10.45	9.92	K	eqns (3), (4), etc.
$\Theta_b \fallingdotseq \theta_{w\,\min}$	11.78	10.61	10.20	9.68	K	eqns (17) and (18)
q_t	9.26×10^5	6.87×10^5	6.13×10^5	5.27×10^5	W m^{-2}	eqn (19)
q_b	8.83×10^5	6.53×10^5	5.81×10^5	5.00×10^5	W m^{-2}	eqns (19)–(22)
τ_t	1.12	1.55	1.76	2.08	s	eqn (20)
τ_b	0.133	0.153	0.161	0.162	s	eqn (21)
$a_1\tau_b^n$	3.87	10.13	12.48	17.68	K	eqn (4)
b_1	4.35	9.25	12.33	22.65	K	eqn (4)
n	0.562	0.328	0.283	0.218		eqn (16)
m	1.56	1.11	1.05	1.02		eqn (8)
K	0.770	0.905	0.922	0.961		eqn (4)
Θ_0	30	50	60	100	K	
Θ_k	25.22	39.07	44.99	68.64	K	
Θ_{wt}	17.71	17.80	17.60	19.39	K	
$\Theta_b \fallingdotseq \theta_{w\,\min}$	17.46	17.43	17.21	18.81	K	
q_t	2.85×10^6	2.84×10^6	2.74×10^6	3.61×10^6	W m^{-2}	
q_b	2.76×10^6	2.72×10^6	2.62×10^6	3.47×10^6	W m^{-2}	
τ_t	0.330	0.333	0.347	0.258	s	
τ_b	0.0341	0.0276	0.0265	0.0159	s	
$a_1\tau_b^n$	7.76	21.64	27.78	49.68	K	
b_1	4.78	10.9	15.0	31.4	K	
n	0.447	0.265	0.225	0.162		
m	1.58	1.16	1.11	1.09		
K	1.39	1.44	1.40	1.42		
Θ_0	50	80	100	140	K	
Θ_k	44.35	66.54	80.71	101.7	K	
Θ_{wt}	29.06	28.05	28.84	26.84	K	
$\Theta_b \fallingdotseq \theta_{w\,\min}$	28.67	27.51	28.26	26.29	K	
q_t	1.18×10^7	1.05×10^7	1.13×10^7	9.20×10^6	W m^{-2}	
q_b	1.15×10^7	1.01×10^7	1.09×10^7	8.87×10^6	W m^{-2}	
τ_t	0.0709	0.0810	0.0746	0.0937	s	
τ_b	0.00616	0.00571	0.00478	0.00515	s	
$a_1\tau_b^n$	15.68	39.0	52.5	75.4	K	
b_1	5.65	13.5	19.3	38.3	K	
n	0.391	0.246	0.209	0.154		
m	1.62	1.20	1.16	1.13		
K	2.59	2.09	1.99	1.67		

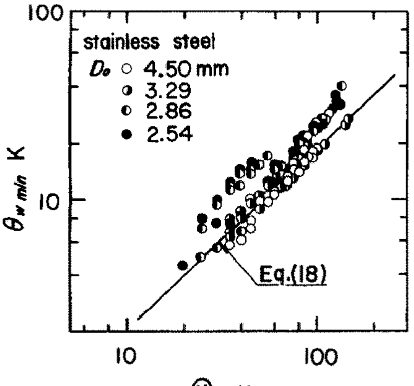

FIG. 5. Relation between $\theta_{w_{\min}}$ and Θ_0 for stainless steel plate.

perature changes are recorded throughout the nucleate boiling and transition boiling regions.

An example of the relation between $\theta_{w_{\min}}$ and Θ_0 is shown in Fig. 5 for a stainless steel and various initial diameters of the droplet. It seems to have a linear relation regardless of D_0. Taking into consideration probable mis-hitting of the junction of the thermocouple by a droplet [1] and also the thermophysical factor X_w of the surface material, we obtain

$$\Theta_0 = 2200 X_w^{-0.729} \theta_{w\min}^{1.03}. \qquad (18)$$

On the other hand the relation between q_t and $\theta_{w\min}$ is shown in Fig. 6. This has similar characteristics to that between q_t and θ_{wt} in the previous paper [1]. We obtain

$$q_t = 800\theta_{w\min}^{2.86} \qquad (19)$$

using about 350 pieces of experimental data.

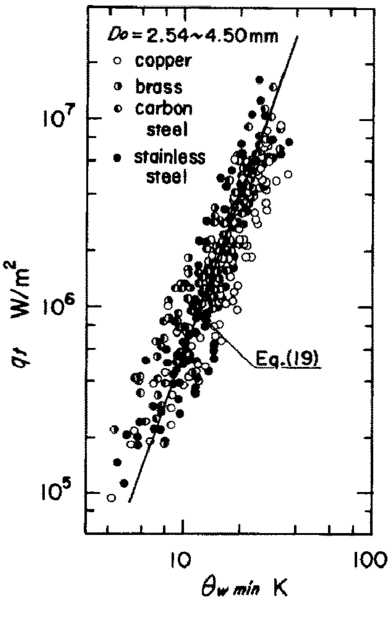

FIG. 6. Time-averaged heat flux q_t vs $\theta_{w\min}$ in the contact period for D_0 = 2.54–4.50 mm and various kinds of surface materials.

Let us use the following correlations published in the previous paper [1]:

$$\tau_t = 1.51 \times 10^{17} X_w^{-2.20} D_0^{3/2} \Theta_0^{-3.01} \qquad (20)$$

$$\tau_b = 4.67 \times 10^{12} X_w^{-2.13} \Theta_0^{-3.35}. \qquad (21)$$

Equation (21) also agrees well with the data of refs. [9–11] (see Fig. 11 in ref. [1]).

2.2. *The relationship among the factors in the present proposal*

$$T_k = (T_{w0}X_R + 20)/(X_R + 1) \qquad (1)$$

$$X_R = X_w/X_L \qquad (2)$$

$$\Theta_w = \Theta_k(1 - Kt^n); \quad 0 \leqslant t < \tau_b \qquad (3)$$

$$\Theta_b = \Theta_k(1 - K\tau_b^n); \quad t = \tau_b \qquad (4)$$

$$\Theta_b \cong \theta_{w\min} \qquad (17)$$

$$\Theta_0 = 2200 X_w^{-0.729} \theta_{w\min}^{1.03} \qquad (18)$$

$$q_t = 800\theta_{w\min}^{2.86} \qquad (19)$$

$$\tau_t = 1.51 \times 10^{17} X_w^{-2.20} D_0^{3/2} \Theta_0^{-3.01} \qquad (20)$$

$$\tau_b = 4.67 \times 10^{12} X_w^{-2.13} \Theta_0^{-3.35} \qquad (21)$$

$$q_w = mq_x = \sqrt{(\pi/3)\{mX_w(a_1t^n + b_1)^2\}}/[t^{1/2}\sqrt{\{b_1^2 + (2a_1b_1t^n)/(n+1) + (a_1^2t^{2n})/(2n+1)\}}]; \quad 0 \leqslant t < \tau_b \qquad (7)$$

$$q_b = \sqrt{(\pi/3)\{mX_w\}} \frac{(a_1\tau_b^n + b_1)^2}{\tau_b^{1/2}\sqrt{\left(b_1^2 + \frac{2a_1b_1\tau_b^n}{n+1} + \frac{a_1^2\tau_b^{2n}}{2n+1}\right)}}; \quad t = \tau_b \qquad (8)$$

$$\frac{q_b\left\{\frac{2n-1}{2n+1}a_1^3\tau_b^{3n} + \frac{5n-3}{n+1}a_1^2b_1\tau_b^{2n} + (4n-3)a_1b_1^2\tau_b^n - b_1^3\right\}}{2\left(b_1^2 + \frac{2a_1b_1\tau_b^n}{n+1} + \frac{a_1^2\tau_b^{2n}}{2n+1}\right)(a_1\tau_b^n + b_1)} = 1.53\Theta_b^{-0.819} na_1\tau_b^n(281\Theta_b^{3.05} - q_b) \qquad (16)$$

where $a_1 = K\Theta_k$, $b_1 = \Theta_0 - \Theta_k$, $\Theta_b = T_b - T_s$, $\Theta_k = T_k - T_s$, $\Theta_w = T_w - T_s$ and $\Theta_0 = T_{w0} - T_s$, also K, m, and n are positive constants in the waiting period.

Let us choose some conditions for the initial surface temperature T_{w0} or Θ_0, and the thermophysical property X_w of the surface material and the initial diameter of the droplet. For example under these conditions, as shown in Table 1, the values of $\theta_{w\min}$, q_t, τ_t and τ_b can be evaluated by equations (18)–(21), respectively. The values of Θ_b can be found approximately by equation (17). On the other hand, as the thermophysical property X_L and saturation temperature T_s of water are already known, T_k or Θ_k can be derived by substituting X_R of equation (2) into equation (1). Thus we can find the values, $a_1\tau_b^n$ or $\Theta_k - \Theta_b$ and b_1, by equation (4).

From Fig. 1 it holds that

$$\int_0^{\tau_b} q_w\,dt + q_b(\tau_t - \tau_b) = q_t\tau_t$$

therefore the following equation is obtained if equations (7) and (8) are substituted into the above equation:

$$q_b = \frac{q_t\tau_t}{\left\{2\left(b_1^2\tau_b + \dfrac{2a_1b_1\tau_b^{n+1}}{n+1} + \dfrac{a_1^2\tau_b^{2n+1}}{2n+1}\right)\Big/(a_1\tau_b^n + b_1)^2 + (\tau_t - \tau_b)\right\}}. \tag{22}$$

It should be noted that this equation does not involve m. Consequently, q_b can be reckoned. Hereupon, in equation (16) all the values except n are known, we can choose n by the trial and error method. Then the values m and K are also found by equations (8) and (4). These evaluated values are shown in Table 1, where the results in the upper, middle and lower columns for each surface material may cover the conditions from nucleate boiling to transition boiling regions for a water droplet on heated surfaces [1].

Now, since a_1, b_1, m, n and K have already been evaluated, as shown in Table 1, it is possible to estimate the time-variation of Θ_w and q_w for $0 < t < \tau_b$ by equations (3) and (7). The relation between q_w and Θ_w is illustrated in Fig. 7. In this figure every ← and × show the direction of time and the condition of the onset of first bubbling after which successive or vigorous boiling holds until the end of the contact. These conditions of the onset might lay on relation (19), because at the onset $t = \tau_b$, $\Theta_w = \Theta_b \cong \theta_{w\min}$ (see equation (17)) and $q_w = q_b \cong q_t$. The latter approximation is reasonable, because the ratio τ_b/τ_t is about one tenth in our experiments [1].

3. DISCUSSION AND CONSIDERATION

3.1. *Assumption of change in surface temperature*, Θ_w

Let us appreciate a penetration depth $\delta_L(t)$ inside a water droplet in the waiting period. If we assume that two semi-infinite solids initially at different temperatures are suddenly brought into contact and Θ_w instantaneously decreases to Θ_k and keeps its temperature thereafter, we can calculate $\delta_L(t)$ by applying equation (6) with $K = n = 0$, though such an assumption leads to an overestimation of $\delta_L(t)$. Thus at the end of the waiting period

$$\delta_L(\tau_b) = \sqrt{(12a_L\tau_b)} = 1.42\times10^{-3}\sqrt{(\tau_b)}.$$

The waiting period τ_b in our experiments is shorter than 0.4 s [1]. So the largest depth, $\delta_L(0.4)$, is about

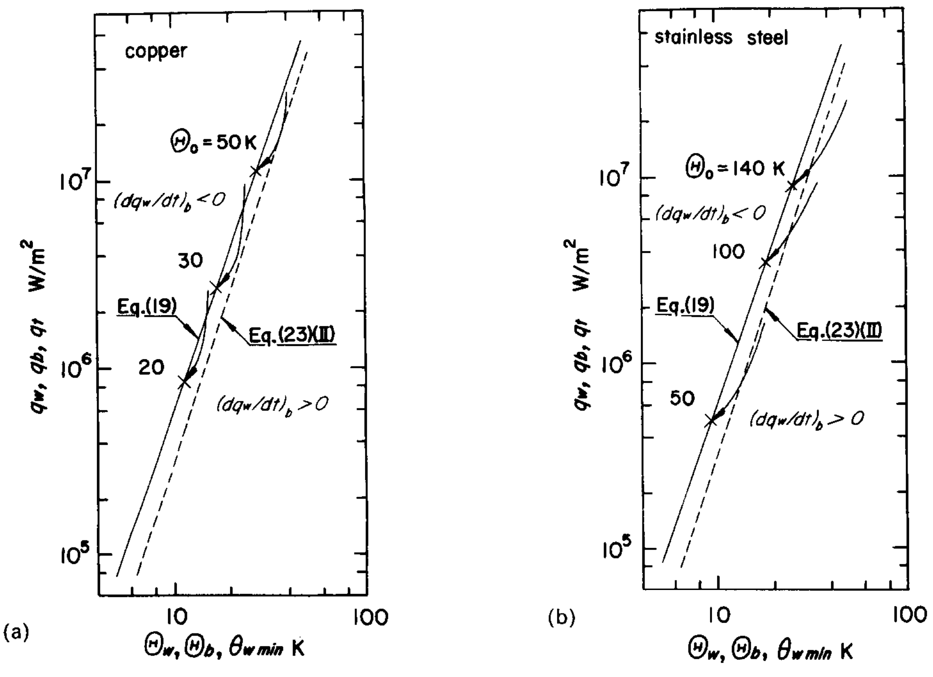

FIG. 7. Relation between q_w and Θ_w in the contact period: (a) copper plate; (b) stainless steel plate.

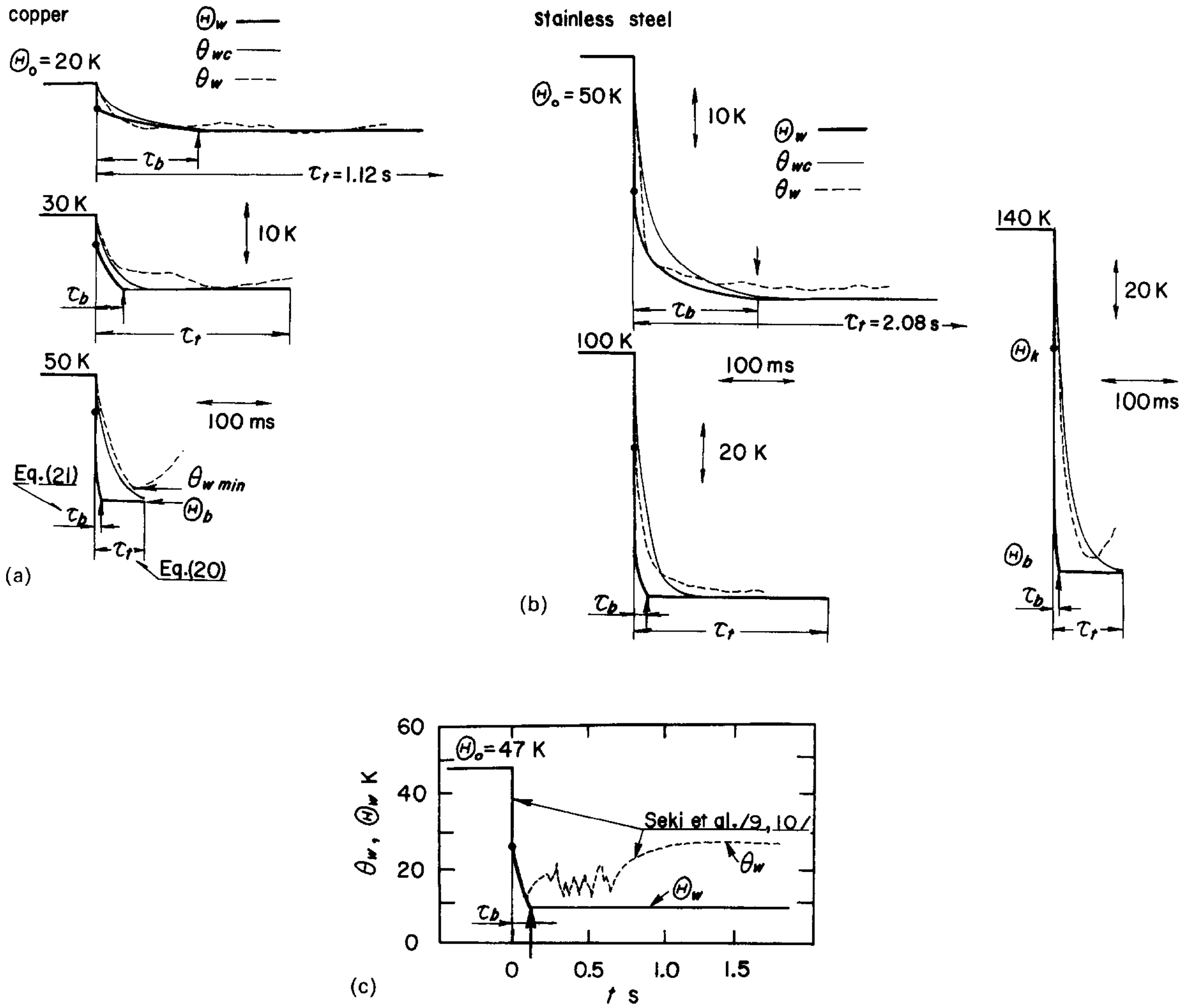

FIG. 8. Examples of comparison among Θ_w, θ_w and θ_{wc}: (a) copper plate; (b) stainless steel plate; (c) stainless steel plate: $\Theta_0 = 47$ K, data taken from refs. [9, 10].

0.9 mm. Hence we may treat a water droplet as a semi-infinite body in the waiting period. At the beginning of the contact there is no convection, however circulation or convection will occur in the droplet thereafter. The intensity of the convection increases together with time t. Accordingly we represent the surface temperature Θ_w by equations (1) and (3).

Generally speaking, the larger the thermophysical factor X_w of the surface material, and the lower the initial surface temperature Θ_0, the slower the surface temperature change near the beginning of the contact is. This is the reason why the exponent n in equation (3) increases with increasing X_w and decreasing of Θ_0, as can be seen in Table 1, where the surface materials are arranged in order of X_w, that is, the X_w of copper being the largest and stainless steel being the smallest. On the other hand the coefficient K increases with X_w at a fixed Θ_0 and also with Θ_0 at a fixed X_w (see also Table 1).

3.2. *The surface temperatures*, Θ_w, θ_{wc} *and* θ_w

When a thermocouple is suddenly thrusted into boiling water, a response of the output could indicate a time constant of the thermocouple. We obtain the time constant τ_0 to be about 15 ms by preliminary examination using the same type and size sheathed thermocouple for measuring the surface temperature.

If we put $\Theta = \Theta_w$, $a_2 = \Theta_0 - \Theta_k$, $K_1 = K\Theta_k$ and $\tau_0 = 0.015$ s in equation (A2) in the Appendix, we can assess the surface temperature $\theta = \theta_{wc}$ which might be recorded by the thermocouple with time constant τ_0.

It is interesting to compare θ_{wc} with θ_w which was found in our experimental works. The relation among Θ_w, θ_{wc} and θ_w are shown in Figs. 8(a) and (b). These θ_{wc} and θ_w seem to be in good agreement with each other. The symbols ● and ↑ are the temperature of Θ_k and the moment of first bubbling, τ_b, which is calculated by equation (21), respectively. On the other hand, a comparison of θ_w measured by a thin-film thermometer [9, 10] with Θ_w and τ_b calculated by the present proposal is shown in Fig. 8(c). Although in this example they [9, 10] do not specify the waiting period τ_b, in the original figure [10] we can find some oscillatory changes, which might indicate nucleate boiling. In this case, it is unnecessary to account for the time constant of the thermometer. We can see that the calculated value Θ_w coincides very well with the experimental value θ_w near the beginning of the contact. Thus, it might be said that the change in surface temperature in the contact period is represented very well by equations (1) and (3).

The time-averaged value of Θ_w can be calculated by its integration between $t = 0$ and τ_r. These values of Θ_{wt} are also listed in Table 1 together with q_t.

3.3. *The condition for the onset of first bubbling*

From equations (7)′ and (16), we can write the following relation at the onset of first bubbling:

$$\begin{aligned}
&\text{(I)} \quad (\mathrm{d}q_w/\mathrm{d}t)_b > 0 \quad \text{for} \quad q_b < 281\Theta_b^{3.05},\\
&\text{(II)} \quad (\mathrm{d}q_w/\mathrm{d}t)_b = 0 \quad \text{for} \quad q_b = 281\Theta_b^{3.05},\\
&\text{(III)} \quad (\mathrm{d}q_w/\mathrm{d}t)_b < 0 \quad \text{for} \quad q_b > 281\Theta_b^{3.05}.
\end{aligned} \qquad (23)$$

Relation (II) in equation (23) is also illustrated in Fig. 7. It is shown that the conditions for the onset (×) of first bubbling are within the upper left-hand side of relation (II), that is, in the region $(\mathrm{d}q_w/\mathrm{d}t)_b < 0$. Since $\mathrm{d}\Theta_w/\mathrm{d}t < 0$ from equation (3)′ and also $\mathrm{d}q_w/\mathrm{d}\Theta_w > 0$ from the arrows in Fig. 7 in the waiting period, it follows that $\mathrm{d}q_w/\mathrm{d}t < 0$ until the onset of first bubbling in the waiting period. This is consistent with the curve of q_w in Fig. 1.

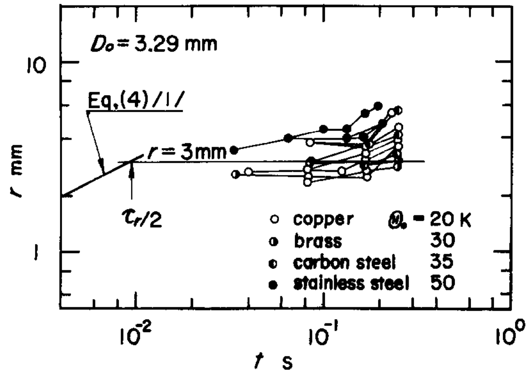

FIG. 9. Examples of the relation between the base radius r and time t.

3.4. *Transient heat transfer coefficient in the waiting period*

Let us calculate the transient heat transfer coefficient α_w from a heated surface to a droplet in the waiting period by our proposal. As mentioned in the former chapter we have obtained the heat flux q_w and T_w as a function of time t, respectively. If we have any information about the relation between the instantaneous base radius r of the droplet in contact with the surface and time t, heat transferred from the surface to the droplet until any time t can be calculated. This heat could only cause a rise in bulk temperature T_B of the droplet from the initial temperature, 20°C, because a waiting period is so short that the evaporation from the droplet might be neglected. Thus we can evaluate the transient heat transfer coefficient α_w as a function of time t in the waiting period by the following equation:

$$\begin{aligned}
\alpha_w &= q_w/(T_w - T_B) = \sqrt{(\pi/3)}\{mX_w(a_1t^n + b_1)^2\}/[t^{1/2}\sqrt{\{b_1^2 + (2a_1b_1t^n)/(n+1) + (a_1^2t^{2n})/(2n+1)\}}]/(T_w - T_B)\\
&= \sqrt{(\pi/3)}\frac{mX_w\Theta_0(1-\Theta_w/\Theta_0)^2}{\sqrt{(t)}(1-\Theta_k/\Theta_0)\left[1+\dfrac{2}{n+1}\dfrac{(\Theta_k-\Theta_w)}{(\Theta_0-\Theta_k)}+\dfrac{1}{2n+1}\left\{\dfrac{(\Theta_k-\Theta_w)}{(\Theta_0-\Theta_k)}\right\}^2\right]^{1/2}(T_w-T_B)} = A_2X_w\Theta_0/\sqrt{(t)} \qquad (24)
\end{aligned}$$

where

$$A_2 = \sqrt{(\pi/3)}\frac{m(1-\Theta_w/\Theta_0)^2}{(1-\Theta_k/\Theta_0)\left[1+\dfrac{2}{n+1}\dfrac{(\Theta_k-\Theta_w)}{(\Theta_0-\Theta_k)}+\dfrac{1}{2n+1}\left\{\dfrac{(\Theta_k-\Theta_w)}{(\Theta_0-\Theta_k)}\right\}^2\right]^{1/2}(T_w-T_B)}.$$

As for the instantaneous base radius r of the droplet, some examples of the time-variation of r in the case of $D_0 = 3.29$ mm taken by a strobo-streak camera and on 8 mm cine camera are shown in Fig. 9. In these examples the temperatures Θ_0 for each surface material correspond to the upper columns in Table 1, that is, close to the lowest limit of the nucleate boiling region or to the longest waiting period τ_b, which is about 0.15 s. Equation (4) in ref. [1] which was obtained for a shorter waiting period in the case of higher surface temperature and the period $\tau_r/2$ for $D_0 = 3.29$ mm at which the base radius took a maximum [1] are also shown. It would be said that the radius r expands within $\tau_r/2$ and then remains constant, 3 mm in the waiting period. The examples of transient heat transfer coefficients α_w are shown in Fig. 10.

When a cold liquid suddenly touches a vertical hot surface which is kept at a constant temperature T_w, a transient heat transfer coefficient α_x at the surface can first be given by purely one-dimensional heat conduction in the liquid before the time τ_c elapsed [15] as follows:

$$\alpha_x = \lambda_L/\sqrt{(a_L\pi t)} = X_L/\sqrt{(t)}; \quad 0 < t < \tau_c \qquad (25)$$

$$\tau_c = 1.80(1.5 + Pr_L)^{1/2}[g\beta_L(T_w - T_B)]^{-1/2}L^{1/2}. \qquad (26)$$

The τ_c represents the time at which purely one-dimensional heat conduction is terminated for each

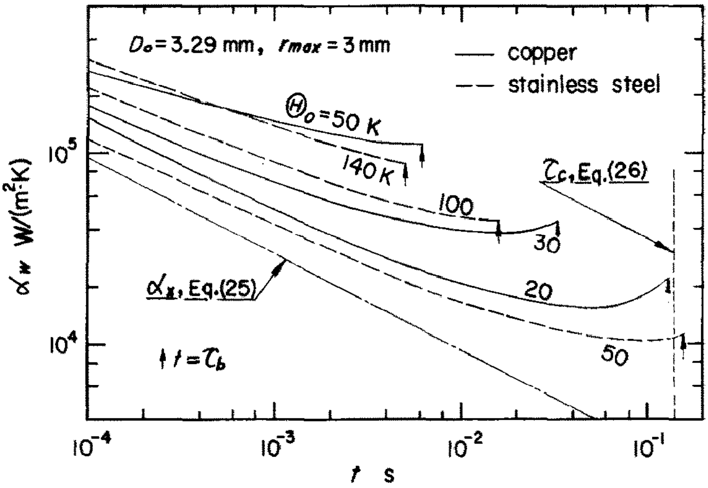

FIG. 10. Examples of heat transfer coefficient in the waiting period.

position L along the vertical plate. If we substitute $T_w - T_B = 220$ K and $L = 0.00329$ m into equation (26), we can get a value of τ_c of about 140 ms. The τ_c for a horizontal plate might not be so different from this value. The α_x and τ_c thus calculated are also shown in Fig. 10.

It can be seen that the α_w are larger than the α_x. This is due to the fact that the heat transfer coefficient increases not only by both convection and circulation within the droplet and the droplet geometry factor like ratio q_w/q_x, but also by Θ_0 and X_w. The latter results from equation (24), because the factor A_2 is scarcely affected by X_w, Θ_0 and t according to the numerical calculation of A_2.

These α_w values are very useful to estimate the cooling ability of a water droplet on various heated surfaces before the onset of the first bubbling. Of course, after the onset of bubbling, the nucleate boiling heat transfer takes place and its coefficient is defined as q_b/Θ_b (see Table 1).

4. CONCLUSION

We have discussed a transient heat transfer to a water droplet on a heated surface in the contact period. Though the present proposal consists of some assumptions which are backed up by experimental and theoretical considerations, a theory of the onset of first bubbling by Michiyoshi [6] and a few experimental works [1, 7, 8] and other works [9–11], calculated values, θ_{wc}, are in good agreement with experimental data, θ_w, in the contact period as shown in Fig. 8. Some examples of the transient heat transfer coefficient in the waiting period evaluated from time-dependent q_w, T_B and r are shown in Fig. 10. After the onset of first bubbling, the nucleate boiling heat transfer takes place and its coefficient is given by q_b/Θ_b with equations (17)–(22) or Table 1.

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APPENDIX

Suppose a thermocouple is used to measure a transient temperature change. The response of a thermocouple of a simple thermal system will be mainly dependent upon the time constant τ_0, when the thermal conductivity of thermometer material is sufficiently large in comparison with its surface conductance. The energy balance in a transient process may be written [14] as follows:

$$\mathrm{d}\theta/\mathrm{d}t = 1/\tau_0(\Theta-\theta)$$

where Θ and θ are the directed and measured temperatures of the thermocouple system, respectively.

Let us consider and develop the relation between Θ and θ as illustrated in Fig. A1, where Θ is changing as follows:

$$\Theta = \Theta_0 - a_2 - K_1 t^n; \quad 0 < t. \tag{A1}$$

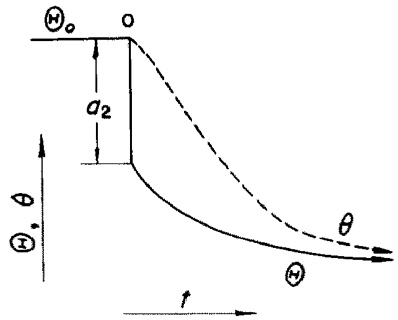

FIG. A1. The relation between the input and output of thermometer with time constant τ_0.

Put initial condition $\Theta = \Theta_0$ at $t = 0$ then

$$\theta = \Theta_0 - a_2(1-\mathrm{e}^{-t/\tau_0}) - (K_1/\tau_0)\mathrm{e}^{-t/\tau_0} \times \left\{\int_0^t (\mathrm{e}^{t/\tau_0}) t^n \, \mathrm{d}t\right\}; \quad 0 < t. \tag{A2}$$

DISCUSSION DU TRANSFERT THERMIQUE ENTRE UNE SURFACE CHAUDE ET UNE GOUTTELETTE D'EAU A LA PRESSION ATMOSPHERIQUE

Résumé—On discute du transfert thermique entre une surface à température variable et une gouttelette d'eau, pendant la période de contact. Le domaine de température initiale de la surface est entre la limite la plus basse de l'ébullition nucléе et le point de Leidenfrost. On s'intéresse aux deux parties de la période de contact: la période avant l'apparition de l'ébullition et la période d'ébullition qui suit immédiatement. Le travail présenté est basé sur quelques considérations théoriques et expérimentales: une théorie de Michiyoshi sur la première ébullition et quelques résultats expérimentaux antérieurement publiés.

TRANSIENTER WÄRMEÜBERGANG AN EINEM WASSERTROPFEN AUF EINER BEHEIZTEN OBERFLÄCHE BEI ATMOSPHÄRENDRUCK

Zusammenfassung—Es wird der transiente Wärmeübergang an einem Wassertropfen auf einer beheizten Oberfläche diskutiert. Dabei wird die Änderung der Oberflächentemperatur während der Kontaktphase, in welcher der Tropfen mit der Oberfläche in Berührung steht, betrachtet. Die anfängliche Oberflächentemperatur liegt zwischen dem Siedebeginn und dem Leidenforst-punkt. Es werden die beiden Phasen der Kontaktperiode unterscheiden: die Verharrungsphase vor der ersten Blasenbildung und die Phase danach. Unsere Berechnung des transienten Wärmeübergangs basiert auf einigen Annahmen, die sich auf theoretische und experimentelle Untersuchungen stützen: eine Theorie von Michiyoshi zum Siedebeginn und einige experimentelle Ergebnisse aus unseren früheren Untersuchungen und andere Arbeiten.

НЕСТАЦИОНАРНЫЙ ТЕПЛООБМЕН КАПЛИ ВОДЫ С НАГРЕВАЕМОЙ ПОВЕРХНОСТЬЮ ПРИ АТМОСФЕРНОМ ДАВЛЕНИИ

Аннотация—Рассматривается нестационарный контактный теплообмен между каплей воды и нагреваемой поверхностью с учетом изменения температуры последней. Начальная температура поверхности изменяется от наименьшего значения при возникновении пузырькового кипения до температуры Лейденфроста. Выделяются два периода контакта: период до начала образования первых пузырьков и период кипения. Предложенная модель нестационарного теплообмена основана на некоторых теоретических и экспериментальных предпосылках, а именно, на теории начального зарождения пузырьков Мичиёши и экспериментальных данных наших предыдущих и других работ.